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Yr12 2013

Multiple choice (1mark each)

Q1) What is the length of the latus rectum for the parabola $x^2 = 2ay$?

a) a b) 2a c) 3a d) 4a

Q2) When does a geometric series have a sum to infinity, when

a) $|r| < 1$ b) $|r| > 1$ c) $|r| = 1$ d) r = 0

Q3) For the quadratic equation $ax^2 + bx + c = 0$, Δ =

a) $\frac{-b}{2a}$ b) $b^2 - 2ac$ c) $b^2 - 4ac$ d) $\sqrt{b^2 - 4ac}$

Q4) If (5,8) lies on the parabola $x^2 = ky$ then the value of k would be

a) 0 b) $\frac{8}{25}$ c) -3 d) $\frac{25}{8}$

Q5) If $x=3$ is a root of $mx^2 - 20x + m = 0$ then the value of the other root is

a) 1 b) 3 c) $\frac{1}{3}$ d) -3

Series(6 marks)

Q1)(2) Find the limiting sum of x^2, x^3, x^4 ..... when $x = \frac{1}{3}$

Q2)(4) The sum of the first five terms of an arithmetic series is four times the fourth term. Also, the sum of the fifth and sixth terms is 65. Find the first term and the common difference.

Quadratic polynomial (26 marks)

Q1)a) (4) Find the value of k for which the equation $x^2 - (k-5)x + (k-7) = 0$ has

i) (1) roots which are reciprocals

ii) (1) roots which are equal in value but opposite in sign

iii) (2) one root equal to -2

b) (2) For the quadratic equation $kx^2 - 6x + k = 0$ what value(s) of k will give real roots ?

Q2) **(3)** A parabola has its vertex at (-3,1) and its focus at (-3,4). Sketch the parabola and indicate on your diagram the directrix and its equation and the equation of the parabola.

Q3) **(3)** Solve $2^{2x} - 12\,(2^x) + 32 = 0$

Q4) **(6)** If α and β are roots of the quadratic equation $2x^2 - 3x - 1 = 0$, find

i) **(1)** $\alpha + \beta$

ii) **(1)** $\alpha\beta$

iii) **(2)** $\alpha^3\beta^2 + \alpha^2\beta^3$

iv) **(2)** $\alpha^2 + \beta^2$

Q5) **(4)** Find the vertex, focal length and axis of the parabola $x = y^2 - 6y + 8$

Q6) **(4)** Consider the parabola $y = x^2 + 2x + 4$ and the tangent to the curve $y = mx$.

(i) **(1)** Show the curves intersect when $x^2 + (2 - m)x + 4 = 0$.

(ii) **(3)** Find the 2 values of m for this line to be a tangent to the curve.

Locus (5 marks)

Q1) **(2)** A point P is equidistant from the line $y = 6$ and the point (0,4), **what** is the equation of the locus of P ?

Q2) **(3)** P moves such that its distance to A(-2,3) is twice its distance to B(1,-1). Find the equation of the locus of P and describe the shape of the locus geometrically.

Integration (20 marks)

Q1) **(5)** Find a primitive of

i) $4x^3$

ii) $-x^5$

iii) $\frac{1}{\sqrt{x}}$

iv) $(2x + 1)^3$

v) $(x^2 - 3)^2$

Q2) **(2)** If $f'(x) = 3x^2 + x - 2$ and the curve passes through (2,1), find the equation of the curve.

Q3) **(4)** Evaluate

i) $\int_0^1 \sqrt{x}\, dx$

ii) $\int_{-2}^{2} x^3 dx$

Q4) **(6)** Given $y = x^2 - 6x + 5$

i) **(1)** Sketch the curve

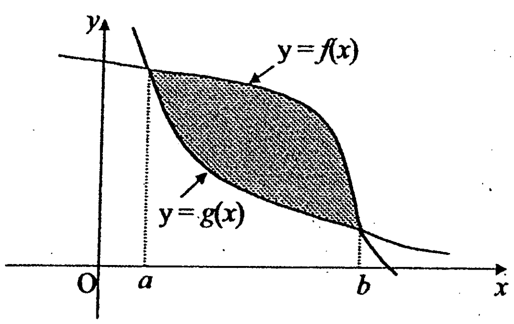
ii) **(2)** Evaluate $\int_1^7 (x^2 - 6x + 5)\, dx$

iii)**(3)** Find the area between the curve and the x axis between $x=1$ and $x=7$,explain why this result is different to the answer in part ii)

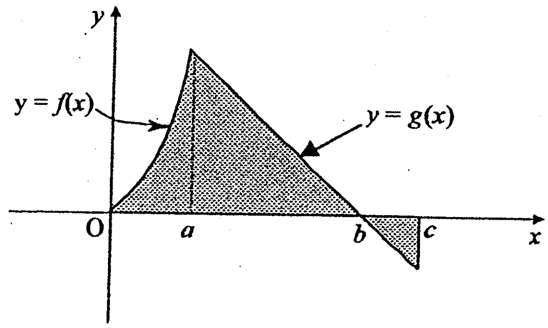
Q5) **(3)**

Write expressions in terms of $f(x)$ and $g(x)$ for the area of the shaded regions in each of the following:

(i)



(ii)



YK 12 2013 2u Suggested sol.

<u>m.c</u> (1) b (2) a (3) c (4) d (5) c 1each

<u>Series</u>

(1) $S_\infty = \dfrac{a}{1-r}$

$= \dfrac{x^2}{1-x}$ — 1

when $x = \frac{1}{3}$

$S_\infty = \dfrac{\frac{1}{9}}{1-\frac{1}{3}} = \dfrac{2}{3}$ — 1

(2) $S_5 = 4 \times T_4$

$\left.\begin{aligned}\tfrac{5}{2}[2a+4d] &= 4[a+3d] \\ 5a+10d &= 4a+12d\end{aligned}\right\}$ — 1

$a = 2d$

$T_5 + T_6 = 65$

$\left.\begin{aligned}a+4d+a+5d &= 65 \\ 2a+9d &= 65\end{aligned}\right\}$ — 1

$13d = 65$

$\left.\begin{aligned}d &= 5 \\ a &= 10\end{aligned}\right\}$ — 2

<u>Quadratic Polynomial</u>

1 a) i) $\dfrac{c}{a} = 1$

$k - 7 = 1$

$k = 8$ — 1

ii) $-\dfrac{b}{a} = 0$

$k - 5 = 0$

$k = 5$ — 1

iii) $x = -2$

$4 + 2(k-5) + k - 7 = 0$ } — 1

$3k = 13$

$k = \dfrac{13}{3}$ — 1

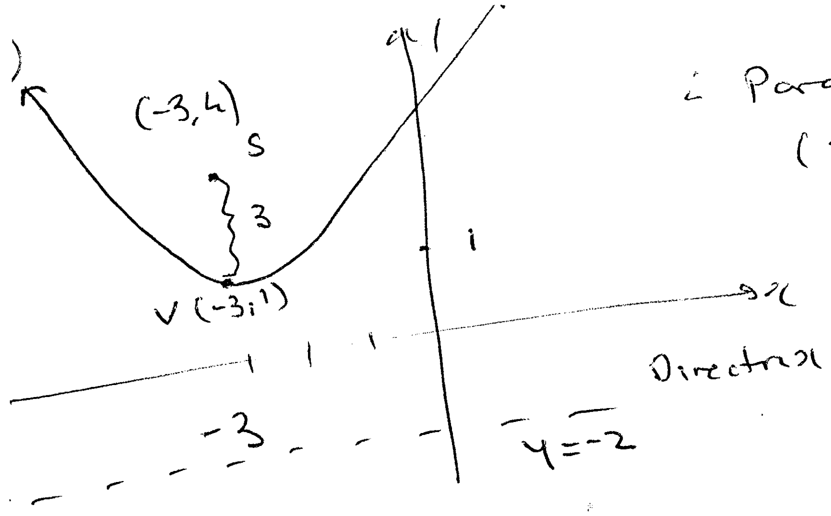
b) $\Delta \geq 0$

$36 - 4k^2 \geq 0$ } — 1

$k^2 \leq 9$

$-3 \leq k \leq 3$ — 1

eqn



$\therefore$ Parabola

$(x+3)^2 = 12(y-1)$

③ $(2^x)^2 - 12(2^x) + 32 = 0$

Let $u = 2^x$ — 1

$u^2 - 12u + 32 = 0$

$(u-4)(u-8) = 0$

$u = 4$ or $u = 8$

$2^x = 4$ or $2^x = 8$ — 2

$x = 2$ $\quad x = 3$

④ i/ $\alpha + \beta = \frac{-b}{a} = \frac{3}{2}$ — 1

ii/ $\alpha\beta = \frac{c}{a} = -\frac{1}{2}$ — 1

iii/ $\alpha^2\beta^2(\alpha+\beta) = \frac{1}{4} \times \frac{3}{2}$ — 1

$= \frac{3}{8}$ — 1

iv/ $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$ — 1

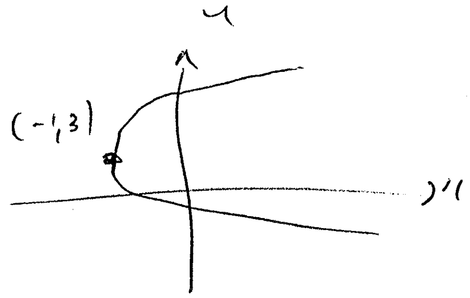
$= \frac{9}{4} + 1$

$= \frac{13}{4}$ — 1

(5) $x - 8 = y^2 - 6y$

$x + 1 = y^2 - 6y + 9$

$(x+1) = (y-3)^2$ — 1



$4a = 1$

$a = \frac{1}{4}$ — 1

$V = (-1, 3)$ — 1

axis is $y = 3$ — 1

(6) i) Intersect when $x^2 + 2x + 4 = mx$

$x^2 + x(2-m) + 4 = 0$ — 1

ii) solution when $\Delta = 0$ — 1

ie $(2-m)^2 - 4\times 4 = 0$

$(2-m)^2 = 16$

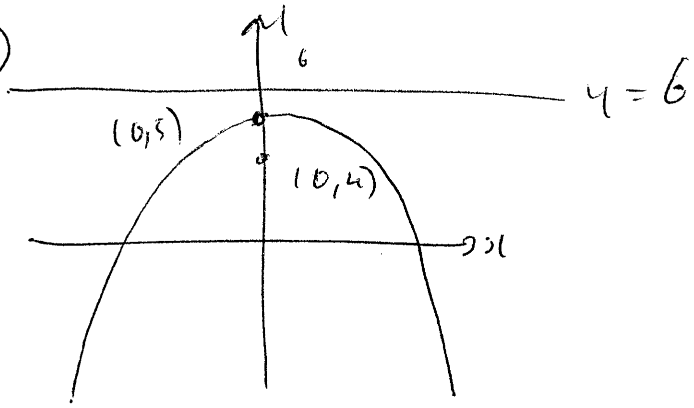
$2 - m = \pm 4$

$-m = -2 \pm 4 = 2$ or -6

$m = -2$ or 6 — 2

Locus

(1)

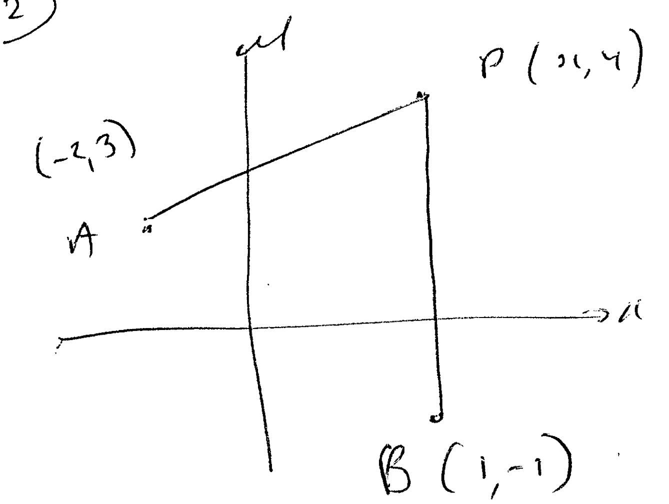


$a = 1$

$x^2 = -4(y-5)$ — 2

(1 if of form $x^2 = -4ay$)

2)



$PA = 2PB$

$PA^2 = 4PB^2$ — 1

$(x+2)^2 + (y-3)^2 = 4(x-1)^2 + 4(y+1)^2$

$3x^2 - 12x + 3y^2 + 14y - 5 = 0$ — 1

This is a circle — 1

Integration

(1) i/ $x^4 + c$ — 1

ii/ $-\frac{x^6}{6} + c$ — 1

iii/ $\int x^{-\frac{1}{2}}\,dx$ — 1

$= 2x^{\frac{1}{2}} + c$ or $(2\sqrt{x} + c)$

Do not penalize if no c

iv/ $\frac{(2x+1)^4}{8} + c$ — 1

v/ $\int (x^4 - 6x^2 + 9)\,dx$ — 1

$= \frac{x^5}{5} - 2x^3 + 9x + c$

(2) $f'(x) = 3x^2 + x - 2$

$f(x) = x^3 + \frac{x^2}{2} - 2x + c$ — 1

Through $(2, 1)$ so $f(2) = 1$

$1 = 8 + 2 - 4 + c$ $\quad c = -5$ — 1

$f(x) = x^3 + \frac{x^2}{2} - 2x - 5$

3) i) $\int_0^1 x^{\frac{1}{2}}\,dx$

$= \left[\frac{2}{3}x^{3/2}\right]_0^1$ — 1

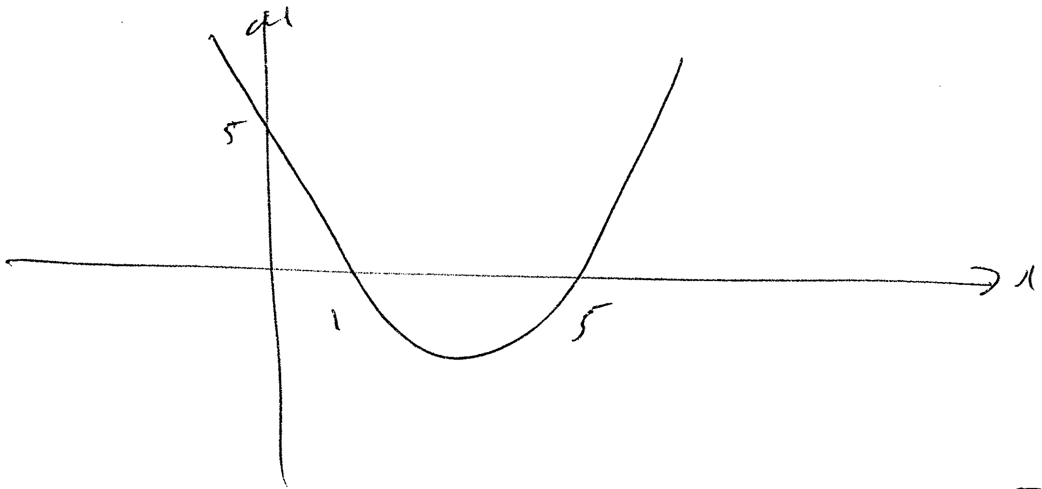
$= \frac{2}{3}$ — 1

ii) $\int_{-2}^{2} x^3\,dx = \left[\frac{x^4}{4}\right]_{-2}^{2}$ — 1

$= \frac{16}{4} - \frac{16}{4}$

$= 0$ — 1

4) i) $y = (x-1)(x-5)$



1

ii) $\int_1^7 (x^2 - 6x + 5)\,dx = \left[\frac{x^3}{3} - 3x^2 + 5x\right]_1^7$ 1

$= \left(\frac{343}{3} - 147 + 35\right) - \left(\frac{1}{3} - 3 + 5\right)$

$= 0$ 1

iii) $A = \int_1^7 (x^2 - 6x + 5)\,dx = \left|\int_1^5 (x^2 - 6x + 5)\,dx\right| + \int_5^7 (x^2 - 6x + 5)\,dx$ 1

$= 21\frac{1}{3}\,u^2$ 1

Different to part ii) as part ii) evaluates the integral & from $x=1 \to x=5$ curve is below x axis so answer is negative. — 1.

(5) i) $A = \int_a^b \{f(x) - g(x)\}\, dx$ — 1

ii) $A = \int_0^a f(x)\,dx + \int_a^b g(x)\,dx - \int_b^c g(x)\,dx$

or $\quad - \quad - \quad + \left| \int_b^c g(x)\,dx \right|$ 2